arXiv:nucl-th/9609049v2 5 Aug 1999

Medium Modification of Nucleon Properties in Skyrme Model

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A Skyrme type Lagrangian for a skyrmion imbedded in a baryon rich environment is proposed. The dependence of static nucleon properties and nucleon - nucleon tensor interaction on nuclear density is investigated.

PACS number(s): 12.39.Dc, 12.39.Fe

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I. INTRODUCTION

Properties of a single nucleon in free space are understood in classical terms by means of a soliton -like solution of nonlinear Lagrangians like the Skyrme Lagrangian [1]. Models have been constructed to consider pion - nucleon and nucleon - nucleon (NN) interactions [2–4] and even to deform the nucleons [5]. An important outstanding problem is the study of these soliton - like structures in a many body system. In heavy ion collision the properties of hadron are to be studied in a hot, non - zero temperature, and dense, density much larger than ρ_o, nuclear matter (here ρ_o is the density of normal nuclear matter). There are good reasons to believe that the properties of hadrons like mass, radii and coupling to external currents change in a hot and dense matter medium. Then the modification of NN interaction has to be understood. The medium plays an important role in changing the strength of the interaction and the relative strengths of central, tensor and spin-orbit interactions.

Quantum Chromodynamics, the fundamental theory of strong interactions, has to be replaced with an effective theory to consider the nuclear interactions in the medium in any consistent manner. In arriving at an effective theory the important constraints of QCD, chiral symmetry and scale invariance have to be retained. Recent considerations of Brown and Rho [6] are an attempt to find and elucidate such a theory. In constructing an effective Lagrangian, $\mathcal{L}_{\text{eff}}$, the in- medium modification is reflected by a change in the vacuum expectation value of a dilatation field. The resulting Lagrangian, which obeys the trace anomaly of QCD almost coincides with Schechter's Lagrangian [7] in form but includes some modified parameters. However this in- medium $\mathcal{L}_{\text{eff}}$ does not take into account the possible modification of the chiral field, since it is considered here as a massless Goldstone boson. On the other hand it is quite natural to assume that, $\mathcal{L}_{\text{eff}}$ has to include the direct distortion of chiral fields. In fact, in a linear approach, Skyrme Lagrangian describes the free pion field and its in - medium modified version must be relevant to pion fields in nuclear matter.

The aim of this paper is to consider a nucleon placed in a nucleus and try to describe this nucleon in the framework of the Skyrme model, taking into account the influence of nucleus as a medium (see also [8,9]). It is not our goal to describe the whole nuclear system. Instead

we shall concentrate on the changes of nucleon properties embedded in nuclei taking into account the influence of baryon rich environment as an external parameter. Our basic idea is that, in the linear approach the $\mathcal{L}_{\rm eff}$ should give the well known [10] equation for the pion field $\partial^\mu\partial_\mu\vec{\pi} + (m_\pi^2 + \hat{\Pi})\vec{\pi} = 0$, where $\hat{\Pi}$ is the polarization function or the self energy of pion field in the medium.

The paper is organized as follows: In Sec. II, we propose a modified Skyrme Lagrangian $\mathcal{L}_{\rm eff}$ - including the distortion of chiral field in the medium. The soliton - like solutions of this Lagrangian represent a skyrmion embedded in the nuclear medium; the Lagrangian is applied to calculate the static properties of the nucleon in Sec. III. In Sec. IV, we consider the possible modifications of nucleon - nucleon tensor interaction due to the presence of the medium. We summarize and discuss the results in Section V.

II. THE IN-MEDIUM SKYRME LAGRANGIAN

The Skyrme model is a theory of nonlinear meson fields where baryons can emerge as soliton solutions. The Skyrme Lagrangian may be written as [1]:

$$\begin{aligned}
\mathcal{L}_{\rm sk} &= \mathcal{L}_2 + \mathcal{L}_{\rm 4a} + \mathcal{L}_{\chi{\rm sb}}, \\
\mathcal{L}_2 &= -\frac{F_\pi^2}{16}\mathrm{Tr}\,(\vec{\nabla}U)\cdot(\vec{\nabla}U^+), \\
\mathcal{L}_{\rm 4a} &= \frac{1}{32e^2}\mathrm{Tr}\,[U^+\partial_i U, U^+\partial_j U]^2, \\
\mathcal{L}_{\chi{\rm sb}} &= -\frac{F_\pi^2}{16}\mathrm{Tr}\,[(U^+-1)m_\pi^2(U-1)],
\end{aligned} \tag{2.1}$$

where in usual notations $F_\pi = 2f_\pi$, e - the Skyrme parameter. The expansion around the vacuum value ($U \approx 1$)

$$U = \exp[2i(\vec{\tau}\vec{\pi})/F_\pi] \approx 1 + \frac{2i}{F_\pi}(\vec{\tau}\vec{\pi}) + \dots \tag{2.2}$$

in (2.1) gives a Lagrangian for free linear pion field:

$$\mathcal{L}_{sk} \approx \mathcal{L}_\pi = -\frac{1}{2}(\vec{\nabla}\vec{\pi})^2 - \frac{1}{2}m_\pi^2\vec{\pi}^2. \tag{2.3}$$

Let us consider a skyrmion inserted in a nucleus. It is well known that pions in nuclei are described [10] by the Lagrangian

$$\mathcal{L}^{\star}_{\pi} = \mathcal{L}_{\pi} - \frac{1}{2}\vec{\pi}\hat{\Pi}\vec{\pi} \tag{2.4}$$

(the asterisk indicates the medium) where $\hat{\Pi}$ is the self energy, or, the polarization operator, which characterizes the modification of the pion propagator in the medium. Bearing in mind an expansion like (2.2) we may generalize (2.1) as:

$$\begin{aligned} \mathcal{L}^{*}_{\text{sk}} &= \mathcal{L}_{2} + \mathcal{L}_{4\text{a}} + \mathcal{L}^{*}_{\chi\text{sb}} \quad , \\ \mathcal{L}^{*}_{\chi\text{sb}} &= -\frac{F_{\pi}^{2}m_{\pi}^{2}}{16}\text{Tr}\left[(U^{+}-1)(1+\hat{\Pi}/m_{\pi}^{2})(U-1)\right] \end{aligned} \tag{2.5}$$

with the only modified term, $\mathcal{L}^{*}_{\chi\text{sb}}$, which describes the distortion of pion field in the medium.

The calculation of the pion self energy in the coordinate space within the Skyrme model in a self consistent way is a special problem. It is not our goal to calculate it in the present paper, since we are not describing the whole system of nucleons in the framework of the Skyrme model. Instead, in the coordinate space we use a simple relation between $\hat{\Pi}$ and the pion-nuclear optical potential $\hat{U}_{opt}$: $\hat{\Pi} \approx 2\omega_{\pi}\hat{U}_{opt}$ [10].

In general, the operator $\hat{\Pi}$ acts on $\vec{R}$ -coordinate of center of mass of soliton as well as on its internal collective coordinate $\vec{r}$ i.e. $\hat{\Pi} = \hat{\Pi}(\vec{R}, \vec{r}-\vec{R})$ [1]. For heavy nuclei the R dependence is weak and for homogenous nucleus it may be neglected totally. So, letting $\hat{\Pi} = 2\omega_{\pi}\hat{U}_{opt}(\vec{r})$ in Eq. (2.5) we may choose an optical potential widely used in the literature. It is clear from (2.5) that when $\hat{U}_{opt}$ is a local one, like "laplacian potential" [10] the modification of the Lagrangian is trivial and mainly consists in changing the pion mass into an effective mass $m^{*}_{\pi} = m_{\pi}\sqrt{1 + 2\hat{U}_{opt}/m_{\pi}}$ in the medium.

Clearly, the most interesting case is to use the nonlocal Kisslinger potential, used both in describing pionic atoms and pion nuclear scattering. At threshold, when $\omega_{\pi} \approx m_{\pi}$, it may be represented in a schematic way [10]:

$$\hat{\Pi} = \chi_{s}(r) + \vec{\nabla}\cdot\chi_{p}(r)\vec{\nabla}, \tag{2.6}$$

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[1] We are indebted to the referee who drawn our attention to this point.

where χ_s and χ_p are some functionals of S- wave and P- wave pion nucleon scattering lengths, and nuclear density - $\rho(r)$. Using (2.6) in (2.5) and bearing in mind integration by part we obtain the following Lagrangian:

$$\begin{aligned}
\mathcal{L}^*_{\text{sk}} &= \mathcal{L}^*_2 + \mathcal{L}_{4\text{a}} + \mathcal{L}^*_{\chi\text{sb}}, \\
\mathcal{L}^*_2 &= -\frac{F_\pi^2}{16}\alpha_p(r)\mathrm{Tr}\,(\vec{\nabla}U)\cdot(\vec{\nabla}U^+), \\
\mathcal{L}_{4\text{a}} &= \frac{1}{32e^2}\mathrm{Tr}\,[U^+\partial_i U, U^+\partial_j U]^2, \\
\mathcal{L}^*_{\chi\text{sb}} &= \frac{F_\pi^2 m_\pi^2}{16}\alpha_s(r)\mathrm{Tr}\,(U+U^+-2),
\end{aligned} \tag{2.7}$$

where $\alpha_p(r) = 1 - \chi_p(r)$, $\chi_p(r)$ - pion dipole susceptibility of the medium, and $\alpha_s(r) = 1 + \chi_s(r)/m_\pi^2$.

Thus, the nonlocal Kisslinger potential modifies not only the pion mass term but also the kinetic term - $\mathcal{L}_2$. Note that in our model the fourth order derivative term - $\mathcal{L}_{4\text{a}}$ remains unchanged. This is not surprising, since this term corresponds to the infinite mass limit of the ρ - meson term [11], whose self energy operator is not considered here. Thus our basic Lagrangian is given in Eq. (2.7) and will be used to investigate the modification of nucleon properties in the medium.

III. THE IN-MEDIUM NUCLEON PROPERTIES

In general, the Lagrangian in Eq. (2.5) is conceivably valid both for finite and infinite nuclei. However in practical calculations in finite nuclei there may arise some difficulties concerned with surface effects and localization of the Skyrmion in nuclear medium. For the simplicity we'll consider medium modifications in homogeneous nuclear matter. In this case $\chi_p(r)$ and $\chi_s(r)$ in the Lagrangian are clearly constants ($\chi_p(r) \equiv \chi_p, \chi_s(r) \equiv \chi_s$) and the skyrmion may be assumed to have spherical symmetry.

For the spherically symmetric static Skyrme ansatz, $U(r) = U_0 = \exp{(i\vec{\tau}\hat{r}\Theta(r))}$,

$\hat{r} = \vec{r}/|r|$, the mass functional for the dimensionless $x = eF_\pi r$ has the form: [2]

$$M_H^* = \frac{4\pi F_\pi}{e}\int\limits_0^\infty dx(\tilde{M}_2^* + \tilde{M}_{4\mathrm{a}} + \tilde{M}^*_{\chi\mathrm{sb}}),$$

$$\begin{aligned} &\tilde{M}_2^* = (\Theta'^2 x^2/2 + s^2)(1-\chi_p)/4, \\ &\tilde{M}_{4\mathrm{a}} = s^2(d/2 + \Theta'^2), \\ &\tilde{M}^*_{\chi\mathrm{sb}} = (1-c)x^2\beta^2(1+\chi_s/m_\pi^2)/4, \end{aligned} \tag{3.1}$$

where $c \equiv \cos(\Theta)$, $s \equiv \sin(\Theta)$, $d = (s/x)^2$, $\beta = m_\pi/(F_\pi e)$. Since the nuclear dipole susceptibility, χ_p, is nearly proportional to the nuclear density ρ, for large densities the $\tilde{M}_2^*$ term, arising from $\mathcal{L}_2^*$ becomes negative and a skyrmion may disappear. Let's discuss this point in detail. The Euler - Lagrange equation for the shape function, $\Theta(x)$, is given as:

$$\Theta''[x^2\alpha_p + 8s^2] + 2\Theta' x\alpha_p + 4\Theta'^2 s_2 - [s_2\alpha_p + 4ds_2 + x^2\beta^2 s\alpha_s] = 0, \tag{3.2}$$

where $s_2 \equiv \sin(2\Theta)$ and a prime corresponds to a derivative with respect to x. As we are not interested in describing the nuclear system as a whole, we use solutions of Eq. (3.2) with $\Theta(0) = \pi$ corresponding to the baryon number $B = 1$. The asymptotic behavior of $\Theta(x)$ at large distances is similar to that for the free case:

$$\begin{aligned} &\lim_{x\to\infty}\Theta(x) = \gamma\frac{(1+\beta^* x)\exp(-\beta^* x)}{x^2}, \\ &\beta^* = \beta\sqrt{\frac{1+\chi_s/m_\pi^2}{1-\chi_p}}. \end{aligned} \tag{3.3}$$

It is well known [10] that for finite nuclei the pion susceptibility is always less than unity, $\chi_p < 1$. However, for infinite nuclear matter with a constant density $\rho = \lambda\rho_o$ ($\rho_o = 0.5m_\pi^3$) there is some critical value of λ when the expression under the square root sign in Eq. (3.3) becomes negative which leads to an exponentional dissipation of soliton solutions. Thus the condition for survival of a skyrmion in the dense matter is equivalent to comparing the

[2]Here the skyrmion is assumed to be placed right in the center of mass of nucleus.

dipole susceptibility with unity as in the usual pion nuclear physics [10]. This result may be compared with the model proposed in Ref. [12], where there are no skyrmion solutions even for real nuclei.

In order to carry out numerical calculations we adopt the following expressions for χ_s and χ_p [10]:

$$\chi_s = -4\pi\eta b_o\rho, \quad \chi_p = \frac{\kappa}{1+g'_o\kappa}, \quad \kappa = 4\pi c_o\rho/\eta, \tag{3.4}$$

where $\eta = 1+m_\pi/M_N$ - a kinematical factor, M_N - mass of the nucleon. The parameters b_o, c_o are some effective pion - nucleon S and P wave scattering lengths, and g'_o - Lorentz-Lorenz or correlation parameter.

We use the following set of empirical parameters $b_o = -0.024 m_\pi^{-1}$, $c_o = 0.21 m_\pi^{-3}$ [13]. Parameters F_π and e have the values $F_\pi = 108 MeV$, $e = 4.84$ as in Ref. [1], so for the free nucleon $M_N = 939 MeV$ and $M_\Delta = 1232 MeV$. Using these values in (3.3) and (3.4) the critical density of nuclear matter ρ_{crit} may be estimated, when a stable solution of Eq. (3.2) (that is a skyrmion) does not exist as $\rho_{crit} \geq 1.3\rho_o$ and $\rho_{crit} \geq 3\rho_o$ ($\rho_o = 0.5 m_\pi^3$-normal nuclear density) for $g'_o = 1/3$ and $g'_o = 0.7$ respectively. Clearly for a real nuclear when $\rho \leq \rho_o$ this model is valid. Standard canonical quantization method [1] gives the familiar expressions for mass of the nucleon and Δ - isobar

$$\begin{aligned} M_N^* &= M_H^* + 3/8\lambda_M^\star, \\ M_\Delta^* &= M_H^* + 15/8\lambda_M^\star, \end{aligned} \tag{3.5}$$

where M_H^* - soliton mass (3.1), $\lambda_M^\star$ is the moment of inertia of the rotating skyrmion:

$$\lambda_M^\star = \frac{8\pi}{3e^3F_\pi}\int_0^\infty dx x^2 s^2[1/4 + \Theta'^2 + d] \tag{3.6}$$

where Θ is the solution of Eq. (3.2). The mass M_H^* may be interpreted as a mass of a soliton of the nonlinear pion fields affected by the medium. Note that, the moment of inertia $\lambda_M^\star$ does not include the nuclear density ρ explicitly. The reason is that the nonstatic parts of the self energy operator are not included in the calculations. Similarly, the isoscalar and isovector mean square radii, defined by zero components of baryon and vector currents, have the same formal expressions as in the free case:

$$< r^2 >^*_{I=0} = -\frac{2}{e^2 F_\pi^2 \pi} \int\limits_0^\infty x^2 \Theta' s^2 dx,$$

$$< r^2 >^*_{I=1} = \frac{1}{e^2 F_\pi^2} \frac{\int\limits_0^\infty x^4 s^2 [1 + 4(\Theta'^2 + d)] dx}{\int\limits_0^\infty x^2 s^2 [1 + 4(\Theta'^2 + d)] dx}. \tag{3.7}$$

Changes in the moment of inertia and size of the nucleon are not crucial, since they are caused only by a modification of the profile function Θ. In contrast, the expression for isovector magnetic moments defined by the space component of the vector current:

$$\mu_{I=1} = \frac{1}{2} \int d\vec{r} \quad \vec{r} \times \vec{V}_3 \tag{3.8}$$

includes medium characteristics explicitly, which arise from the contributions of the kinetic term $\mathcal{L}_2^*$ to the vector current

$$\vec{V}_k = -i \frac{F_\pi^2}{16} (1 - \chi_p) \mathrm{Tr}\, \vec{\tau} (L_k + R_k) + \frac{i}{16 e^2} \mathrm{Tr}\, \vec{\tau} \{ [L_\nu [L_k, L_\nu]] + [R_\nu [R_k, R_\nu]] \} \tag{3.9}$$

where $L_\mu = U^+ \partial_\mu U$, $R_\mu = U \partial_\mu U^+$. Hence for the nucleon in nuclei simple relations between magnetic moments and momentum of inertia such as $\mu^p_{I=1} = \lambda_M/3$ shown in Ref. [1] do not work. Table I illustrates modifications of the static properties of the nucleon in infinite nuclear matter. Here the classical value of the correlation factor $g'_o = 1/3$ is used.

Early arguments [14] about changes of the nucleon size in the medium were, in part, based on the expectation that $r^*/r = M_N/M_N^*$, where r^* and M_N^* are the nucleon radius and mass within the nuclear medium, and r, M_N are the same two quantities for a free nucleon. As it is clear from Table I, in the present model the renormalization of the nucleon mass is much larger than the renormalization of the nucleon radius. The renormalization of the nucleon radii in Eq. (3.7) has been caused only by a modification of the profile function $\Theta(r)$ (see Fig.1) in the nucleus, while the modification of M_N is caused in addition by the factor $(1 - \chi_p)$ in Eq. (3.1).

There are no direct experimental values of static properties of a nucleon bound in nuclei. In contrast many theoretical approaches are proposed to estimate them. Many of them deal with an explanation of the EMC effect. For example, in the nuclear binding model [15],

$M_N^* = 700 MeV$ (appropriate for Fe) and $M_N^* = 600 MeV$ (appropriate for Au) have been found. On the other hand a calculation of the nucleon effective mass M_N^* is an important problem in quantum hadrodynamics (QHD). The recent results obtained by including π, ρ, ω - meson fields explicitly in the Lagrangian of QHD give $M_N^* \approx 620 MeV$ at zero temperature for $\rho = \rho_o$ [16]. In comparison with these results, our model gives $M_N^* = 572 MeV$ for normal nuclear matter for $g_o' = 1/3$. Note that, our results are very sensitive to the value of Lorentz-Lorenz parameter g_o'. For example, using another value, $g_o' = 0.4$, one may get $M_N^* = 596 MeV$. This fact is illustrated in Fig.2a, where the dependence of M_N^* on g_o' is plotted for $\rho = 0$, $\rho = 0.5\rho_o$ and $\rho = \rho_o$ using solid, dotted and dashed curves respectively. The present approach is similar in some sense to the soliton model of Ref. [17] where the mean field approximation for Friedberg-Lee approach is used. A swelling of the nucleon size $\sim 30\%$ predicted there, is in good agreement with our result (see Table I). On the other hand there is a pion excess model proposed by M. Ericson [18], which explains the swelling by a distortion of the pion cloud in the medium. However, she obtained a very large modification of the nucleon size, i.e. nearly doubling of the free value of the r.m.s. radius. In addition in the pion excess model the effect of swelling concerns only the isovector radius, whereas in this approach the swelling includes both isovector and isoscalar radii.

Another interesting phenomenon of pion nuclear physics is that, in nuclei the axial coupling constant g_A, governing Gamow - Teller transitions, may be modified significantly from its free-space value $g_A \approx 1.25$. It is shown that g_A is systematically renormalized downward in finite nuclei [19]. A most remarkable observations, made in Ref. [20] based on a model independent analysis of β - decay and magnetic moment data of the mirror nuclei, $(3 \leq A \leq 39)$, is that the axial coupling constant in nuclei equals unity to a very good accuracy: $g_A^* = 1.00 \pm 0.02$, that is $g_A^*/g_A = 0.8$ for nuclear matter.

Although the Skyrme model, especially in its original version, gives an underestimate for the value of g_A ($g_A = 0.65$ for the free case) we may try to investigate the quenching phenomenon within the present approach. It is easy to show that the expression for g_A is the same as in the free case but there is an additional factor in the term arising from the kinetic term:

$$g_A{}^* = -\frac{\pi}{3e^2}\int_0^\infty dx\, x^2 (g_2^*(x) + g_4(x)),$$

$$g_2^*(x) = (1-\chi_p)\cdot(\Theta' + s_2/x), \tag{3.10}$$

$$g_4(x) = 4[s_2(\Theta'^2 + d)/x + 2\Theta' d].$$

In nuclei ($\chi_p < 1$) g_A decreases due to the factor $(1-\chi_p)$ under the integral in Eq. (3.10). The decrease of g_A reaches 38% for nuclear matter ($\rho = \rho_o$ with $g_o' = 1/3$) (Table I). This is consistent with the estimates carried out in Δ - hole coupling model using the random phase approximation: $g_A{}^*/g_A \approx 0.67$ for $\rho/\rho_o = 1$ and $g_A{}^*/g_A \approx 0.8$ for $\rho/\rho_o = 1/2$. The g_o' dependence of $g_A{}^*/g_A$ is shown in Fig.2b. This dependence is in qualitative agreement with the formula $g_A{}^*/g_A = [1 - 4g_o' L(0)/9]^{-1}$ presented in the review article [19].

For the nuclear with constant density the Lagrangian in Eqs. (2.7) with a representation of the polarization operator in Eqs. (2.6), (3.4) has only 2 parameters dictated by pion nucleon scattering. Here the effective pion - nucleon scattering lengths have been used. However in nuclear matter the pion field is localized very close to nucleons in contrast with the case of pionic atoms. One may ask if the present model is able to make predictions about effective scattering lengths b_o and volumes c_o in nuclear matter? To do this we have to compare our results with experimental data. The ratio, $g_A{}^*/g_A$, is well established to be 0.8, while the pionic data analysis yields a value of 0.62 (see the last line of Table I). In the nuclear matter c_o is reduced by a factor of 2 almost independent of the value of g_o' to get the correct quenching (Table II). For this optimal case the effective nucleon mass is also close to the common value of 700 MeV. In addition $g_A{}^*/g_A$ is not sensitive to the S - wave scattering length b_o. A reduction of the effective P - wave scattering length in nuclei may be clearly understood by the fact that quenching of g_A is equivalent to a reduction of the pion - nucleon coupling constant $g_{\pi NN}$ and hence the pion - nucleon amplitude.

IV. IN - MEDIUM NN TENSOR INTERACTION

Not only the static properties of hadrons but also the dynamical ones are modified by the

presence of the medium. In - medium NN interaction differs from the corresponding one in free space due to Pauli blocking (which is not considered here) and due to the modification of propagators of exchanged mesons [21]. We investigate the nucleon - nucleon interaction potential by using the product approximation:

$$U(\vec{x};\vec{r_1},\hat{A}_1;\vec{r_2},\hat{A}_2) = \hat{A}_1 U_0(\vec{x}-\vec{r_1})\hat{A}_1^{+}\hat{A}_2 U_0(\vec{x}-\vec{r_2})\hat{A}_2^{+} \equiv U_1 U_2, \tag{4.1}$$

where $U_0(\vec{x}-\vec{r_i})$ for $i=1,2$ is the hedgehog solution $(U_0(\vec{r}) = \exp{(i\vec{\tau}\hat{r}\Theta(r))},\ \hat{r} = \vec{r}/|r|)$ located at $\vec{r_i}$, and A_i is the collective coordinate to describe the rotation. The in - medium NN interaction may be defined by:

$$V_{NN}(\vec{r}) = -\int d\vec{x}[\mathcal{L}^{*}_{\rm sk}(U_1U_2) - \mathcal{L}^{*}_{\rm sk}(U_1) - \mathcal{L}^{*}_{\rm sk}(U_2)], \tag{4.2}$$

where $\vec{r}$ is the relative coordinate between two skyrmions $(\vec{r} = \vec{r_1} - \vec{r_2})$. The static NN potential may be obtained by using a standard technique [2] which gives the following general representation:

$$V_{NN}(\vec{r}) = V_C(r) + (\vec{\tau}_1\vec{\tau}_2)(\vec{\sigma}_1\vec{\sigma}_2)V_{\sigma\tau}(r) + (\vec{\tau}_1\vec{\tau}_2)S_{12}V_T(r) \tag{4.3}$$

Unfortunately, the original Skyrme model for the free case cannot describe the intermediate range attraction in the central potential within this approximation [2]. This may be improved by the inclusion of a scalar σ - meson in the Lagrangian [3,4], which is not taken into account in the present calculations. Here, it is more interesting for us to consider the tensor part, $V_T(r)$, of V_{NN} in Eq. (4.3), caused mainly by the exchange of pions, modified in the medium. This part of V_{NN} plays an important role in spin -isospin excitations and pion-like excited states in nuclear physics.

Actually for finite nucleus the product ansatz (4.1) should be modified taking into account nonspherical effects. But for homogenous nuclear matter it is as valid as in the case of free space especially at intermediate and large separations. In fact, formally in this approach, the main difference between the in - medium case ($\rho \neq 0$) and the free one ($\rho = 0$) is that the contribution to the potential arising from $\mathcal{L}_2$ and $\mathcal{L}_{\chi\rm sb}$ should be multiplied by factors of $(1-\chi_p)$ and $(1+\chi_s)$ respectively. The resulting $V_T(r)$ is presented in Fig.3 for normal nuclear matter densities $\rho = 0$, $\rho = 0.5\rho_o$ and $\rho = \rho_o$ (solid, dotted and dashed curves respectively).

The parameters of the optical potential in Eq. (3.4) were chosen so as to reproduce the relation ${g_A}^*/g_A = 0.8$ for nuclear matter (II-line of Table II). The nucleon - nucleon tensor interaction in a nucleus appears to be weaker than it is in free space ($\rho = 0$). This suppression of $V_T(r)$ in nuclear matter was shown by Hosaka and Toki [22] using normalized exchange meson masses in accordance with the scaling model of Brown and Rho [6]. For finite nuclei it was shown in Ref. [23] by analyzing the energy difference of $T = 1$ and $T = 0$, $J = 0$ states in ^{16}O.

V. DISCUSSION AND SUMMARY

We have proposed a modified Lagrangian $\mathcal{L}^*_{\rm sk}$ for a skyrmion placed in a nuclear medium. In constructing the Lagrangian we required that in the linear approach (2.2) it would yield the well known equation for the pion field: $\partial^\mu\partial_\mu\vec{\pi} + (m_\pi^2 + \hat{\Pi})\vec{\pi} = 0$. Having been satisfied by inclusion of the pion self energy $\hat{\Pi}$ in to the free space Skyrme Lagrangian, this requirement determined the explicit coordinate dependence of $\hat{\Pi}$ in Eq. (2.5). Actually, for a moving Skyrmion with $U = U(\vec{r} - \vec{R})$ the similar dependence $\hat{\Pi} = \hat{\Pi}(\vec{r} - \vec{R})$ should be fixed. Otherwise the above equation would not be consistent with the medium modified Lagrangian in Eq. (2.5).

A much more general choice as $\hat{\Pi} = \hat{\Pi}(\vec{R}, \vec{r} - \vec{R})$, which is essential for a finite nuclear, should have given a chance to get an information about energy levels of the bound skyrmion as well as about in - medium modification of its internal parameters (mass, size etc.) Since the latter is one of the most exiting topics of nuclear physics, especially on the light of forthcoming ultrarelativistic heavy - ion experiments (e.g. at RHIC), we restricted ourselves with the study of in medium changes in homogeneous nuclear matter, where the coordinate dependence of $\hat{\Pi}$ is simpler.

As an input data, apart from F_π and e, the present approach uses the nuclear density and effective pion - nucleon scattering lengths. In baryon number one sector, $B = 1$, the in - medium nucleon properties can be estimated. Within the present modified Skyrme model it is also easy to study the in - medium nucleon - nucleon interaction by using standard product approximation in the sector with $B = 2$. Consideration of other sectors with $B > 2$

has no sense, since the description of even light nuclear in framework of Skyrme model is a long standing problem.

Let us recall here our main results:

(i) The critical nuclear density ρ_{crit}, where a skyrmion, hence, a nucleus remains stable is $\rho_{crit} \leq 1.3\rho_o$ and $\rho_{crit} \leq 3\rho_o$ for $g'_o = 1/3$ and for $g'_o = 0.7$ respectively. This fact shows the strong dependence of ρ_{crit} on the Landau parameter g'_o.

(ii) The in - medium effects such as the swelling of a nucleon and decrease of its mass are not as large as predicted by pion excess models. The in - medium change of nucleon mass occurs mainly due to the modification of the second derivative term $\mathcal{L}_2$ and depends on the size of c_o - isoscalar P- wave pion nucleon scattering volume.

(iii) A study of the quenching effect of axial coupling constant, g_A, in the nuclear matter showed that effective c_o is much smaller than that predicted by the pionic atom analysis.

(iv) These modifications can occur naturally in the NN interaction. Particularly the tensor part of the interaction in nuclei appears to be weaker than in free space.

Modification of nucleon properties found in present paper are understood by means of medium effects on the chiral nonlinear field and consequently on the shape and mass of the soliton.

Another explanation of the in - medium modifications, based on scale invariant arguments, has been recently proposed by Brown and Rho [6]. The authors implemented the (broken) scale invariance of QCD in the Skyrme model and suggested that changes in hadron properties might arise from a universal scaling related to the scaling anomaly of QCD. However, further analyses [24] have shown that these changes must be small due to the large mass of dilaton, associated here with a glueball.

In a more fundamental level the origin of these changes is hidden in a partial restoration of chiral symmetry i.e. in decrease of quark condensate in nuclear matter [6,24]. Unfortunately, there are no quark degrees of freedom in the Skyrme model. So, in the framework of this model it is natural to believe that the modification of nucleon properties in the medium are caused by the influence of the latter to the nonlinear pion fields. This influence can be taken

into account , for example, in terms of pion self energy and, in general, would not be covered only by a trivial scale renormalization of parameters of the model.

ACKNOWLEDGMENTS

We thank M. Birse, A. Mann, V. Petrov and M. Rho for useful discussions. M.M. Musakhanov and A.M. Rakhimov are indebted to the University of Alberta for hospitality during their stay, where the main part of this work was performed. The research of F. Khanna is supported in part by National Science and Engineering Research Council of Canada.

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TABLES

TABLE I. Ratio of the static properties of the nucleon in the medium (denoted by asterics) to that of the free nucleon for various values of the nuclear density $\rho = \lambda \cdot 0.5 m_\pi^3$ ($g_o' = 1/3$).

λ	$\frac{M_N^*}{M_N}$	$\frac{g_A^*}{g_A}$	$\sqrt{\frac{<r^2>^*_{M,I=0}}{<r^2>_{M,I=0}}}$	$\sqrt{\frac{<r^2>^*_{I=0}}{<r^2>_{I=0}}}$	$\sqrt{\frac{<r^2>^*_{I=1}}{<r^2>_{I=1}}}$	$\sqrt{\frac{<r^2>^*_{p}}{<r^2>_{p}}}$	$\sqrt{\frac{<r^2>^*_{n}}{<r^2>_{n}}}$
0.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.788	0.845	1.121	1.170	1.060	1.095	0.969
0.75	0.696	0.743	1.177	1.255	1.086	1.140	0.939
1.00	0.609	0.619	1.230	1.344	1.113	1.188	0.900

TABLE II. Nucleon effective mass -M_N^* and modification of the nucleon size in normal nuclear matter ($\rho = \rho_o = 0.5 m_\pi^3$). The effective pion- nucleon scattering lengt -b_o and scattering volume -c_o are chosen so that ${g_A}^*/g_A = 0.8$.

g_0'	b_o (m_π^{-1})	c_o (m_π^{-3})	M_N^* (MeV)	$\sqrt{\frac{<r^2>^*_{p}}{<r^2>_{p}}}$
1/3	-0.024	0.125	719.	1.089
0.6	-0.024	0.150	714.	1.092
0.6	0.0	0.140	680.	1.10

FIGURE CAPTIONS

FIG. 1. The profile function $\Theta(r)$ of a free skyrmion (solid curve) and that of a skyrmion in the nuclear matter $\rho = 2.5\rho_o$ (dashed curve). Here $g'_o = 0.7$.

FIG. 2(a). The dependence of the effective nucleon mass on Lorentz - Lorenz parameter g'_o. Solid, dotted and dashed curves are for $\rho = 0$, $\rho = 0.5\rho_o$ and $\rho = \rho_o$ respectively.

FIG. 2(b). The same as in Fig.2(a), but for the ratio g_A^*/g_A.

FIG. 3. The tensor part of the NN potential - $V_T(r)$. Solid, dotted and dashed curves are for $\rho = 0$, $\rho = 0.5\rho_o$ and $\rho = \rho_o$ respectively. Here $g'_o = 0.6, b_o = -0.024 m_\pi^{-1}$ and $c_o = 0.15 m_\pi^{-3}$.